Synopsis

The analysis of wave propagation in hyperelastic waveguides has significant applications in various branches of engineering like Non-Destructive Testing and Evaluation, impact analysis, material characterization and damage detection. Linear elastic models are typically used for wave analysis since they are sufficient for many applications. However, certain solids exhibit inherent nonlinear material properties that cannot be adequately described with linear models. In the presence of large deformations, geometric nonlinearity also needs to be incorporated in the analysis. These two forms of nonlinearity can have significant consequences on the propagation of stress waves in solids. A detailed analysis of nonlinear wave propagation in solids is thus necessary for a proper understanding of these phenomena.

The current research focuses on the development of novel algorithms for nonlinear finite element analysis of stress wave propagation in hyperelastic waveguides. A full three-dimensional (3D) finite element analysis of stress wave propagation in waveguides is both computationally difficult and expensive, especially in the presence of nonlinearities. By definition, waveguides are solids with special geometric features that channel the propagation of stress waves along certain preferred directions. This suggests the use of kinematic waveguide models that take advantage of the special geometric features of the waveguide. The primary advantage of using waveguide models is the reduction of the problem dimension and hence the associated computational cost. Elementary waveguide models like the Euler-Bernoulli beam model, Kirchoff plate model etc., which are developed primarily within the context of linear elasticity, need to be modified appropriately in the presence of material/geometric nonlinearities and/or loads with high frequency content. This modification, besides being non-trivial, may be inadequate for studying nonlinear wave propagation and higher order waveguide models need to be developed. However, higher order models are difficult to formulate and typically have complex governing equations for the kinematic modes. This reflects in the relatively scarce research on the development of higher order waveguide models for studying nonlinear wave propagation. The formulation is difficult primarily because of the complexity of both the governing equations and their linearization, which is required as part of a nonlinear finite element analysis. One of the primary contributions of this thesis is the development and implementation of a general, flexible and efficient framework for *automating* the finite element analysis of higher order kinematic models for nonlinear waveguides. A hierarchic set of higher order waveguide models that are compatible with this formulation are proposed for this purpose. This hierarchic series of waveguide models are similar in form to the kinematic assumptions associated with standard waveguide models, but are different in the sense that no conditions related to the stress distribution specific to a waveguide are imposed since that is automatically handled by the proposed algorithm. The automation of the finite element analysis is accomplished with a dexterous combination of a nodal degrees-of-freedom based assembly algorithm, automatic differentiation and a novel procedure for numerically computing the finite element matrices directly from a given waveguide model. The algorithm, however, is quite general and is also developed for studying nonlinear plane stress configurations and inhomogeneous structures that require a coupling of continuum and waveguide elements. Significant features of the algorithm are the automatic numerical derivation of the finite element matrices for both linear and nonlinear problems, especially in the context of nonlinear plane stress and higher order waveguide models, without requiring an explicit derivation of their algebraic forms, automatic assembly of finite element matrices and the automatic handling of natural boundary conditions. Full geometric nonlinearity and the hyperelastic form of material nonlinearity are considered in this thesis. The procedures developed here are however quite general and can be extended for other types of material nonlinearities. Throughout this thesis, it is assumed that the solids under investigation are homogeneous and isotropic.

The subject matter of the research is developed in four stages: First, a comparison of different finite element discretization schemes is carried out using a simple rod model to choose the most efficient computational scheme to study nonlinear wave propagation. As part of this, the frequency domain Fourier spectral finite element method is extended for a special class of weakly nonlinear problems. Based on this comparative study, the Legendre spectral element method is identified as the most efficient computational tool. The efficiency of the Legendre spectral element is also illustrated in the context of a nonlinear Timoshenko beam model. Since the spectral element method is a special case of the standard nonlinear finite element Method, differing primarily in the choice of the element basis functions and quadrature rules, the automation of the the standard nonlinear finite element method is undertaken next. The automatic finite element formulation and assembly algorithm that constitutes the most significant contribution of this thesis is developed as an efficient numerical alternative to study the physics of wave propagation in nonlinear higher order structural models. The development of this algorithm and its extension to a general automatic framework for studying a large class of problems in nonlinear solid mechanics forms the second part of this research. Of special importance are the automatic handling of nonlinear plane stress configurations, hierarchic higher order waveguide models and the automatic coupling of continuum and higher order structural elements using specially designed transition elements that enable an efficient means to study waveguides with local inhomogeneities. In the third stage, the automatic algorithm is used to study wave propagation in hyperelastic waveguides using a few higher order 1D kinematic models. Two variants of a particular hyperelastic constitutive law – the 6-constant Murnaghan model (for rock-like solids) and the 9-constant Murnaghan model (for metallic solids) – are chosen for modeling the material nonlinearity in the analysis. Finally, the algorithm is extended to study energy-momentum conserving time integrators that are derived within a Hamiltonian framework, thus illustrating the extensibility of the algorithm for more complex finite element formulations.

In short, the current research deals primarily with the identification and automation of finite element schemes that are most suited for studying wave propagation in hyperelastic waveguides. Of special mention is the development of a novel unified computational framework that automates the finite element analysis of a large class of problems involving nonlinear plane stress/plane strain, higher order waveguide models and coupling of both continuum and waveguide elements. The thesis, which comprises of 10 chapters, provides a detailed account of various aspects of hyperelastic wave propagation, primarily for 1D waveguides.